CH2

Equilibrium of aparticle

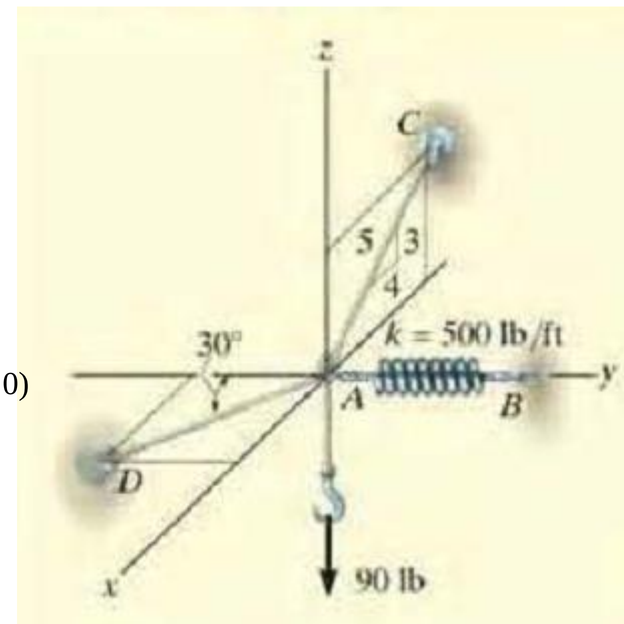
The equations of the equilibrium are

$$\Sigma \mathbf{F} = \mathbf{0} \qquad \text{or} \qquad \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases}$$

Example 1

A 90-lb load is suspended from the hook shown in Fig. If the load is supported by two cables and a spring having a stiffness $k = 500$ lb/ft, determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x–y plane and cable AC lies in the x–z plane.

Solution



$$\vec{F}_B = F_B\vec{j} = (0, F_B, 0)$$
$$\vec{F}_C = F_C(-\frac{4}{5}\vec{i} + \frac{3}{5}\vec{k}) = (-\frac{4}{5}F_C, 0, \frac{3}{5}F_C)$$
$$\vec{F}_D = F_D(\sin 30\vec{i} - \cos 30\vec{j}) = (F_D \sin 30, -F_D \cos 30, 0)$$
$$\vec{W} = (0, 0, -90)$$

$\Sigma F_x = 0;$	$F_D \sin 30° - \left(\frac{4}{5}\right)F_C = 0$	(1)
$\Sigma F_y = 0;$	$-F_D \cos 30° + F_B = 0$	(2)
$\Sigma F_z = 0;$	$\left(\frac{3}{5}\right)F_C - 90\text{ lb} = 0$	(3)

Solving Eq. (3) for F_C, then Eq. (1) for F_D, and finally Eq. (2) for F_B, yields

$$F_C = 150 \text{ lb} \qquad \textit{Ans.}$$

$$F_D = 240 \text{ lb} \qquad \textit{Ans.}$$

$$F_B = 207.8 \text{ lb} \qquad \textit{Ans.}$$

The stretch of the spring is therefore

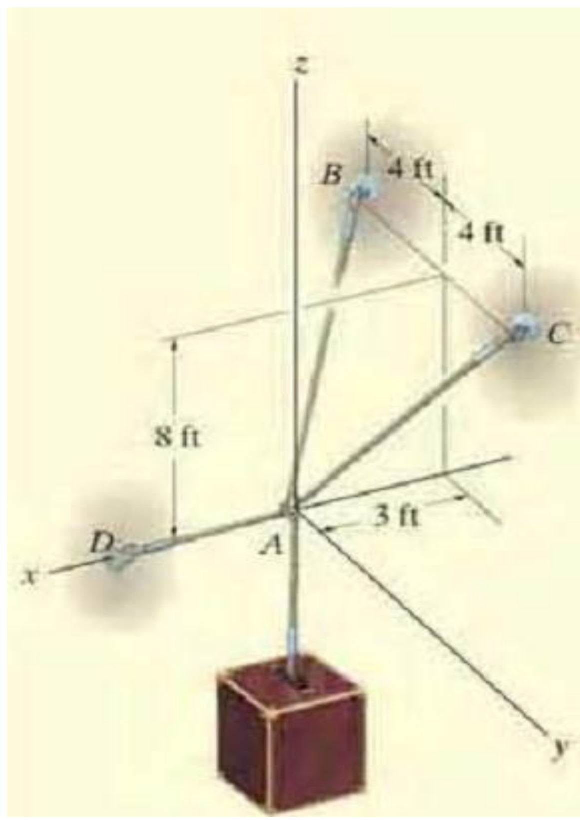
$$F_B = k s_{AB}$$

$$207.8 \text{ lb} = (500 \text{ lb/ft})(s_{AB})$$

$$s_{AB} = 0.416 \text{ ft} \qquad \textit{Ans.}$$

Example 2

Determine the force in each cable used to support the 40-lb crate shown in Fig



Solution

$$\mathbf{F}_B = F_B\left[\frac{-3\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}}\right]$$

$$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\left[\frac{-3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (4)^2 + (8)^2}}\right]$$

$$= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\mathbf{i}$$

$$\mathbf{W} = \{-40\mathbf{k}\} \text{ lb}$$

Equilibrium requires

$\Sigma \mathbf{F} = \mathbf{0};$ $$\mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$

$$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$
$$-\ 0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = \mathbf{0}$$

Equating the respective **i**, **j**, **k** components to zero yields

$\Sigma F_x = 0;$ $$-0.318F_B - 0.318F_C + F_D = 0 \quad (1)$$

$\Sigma F_y = 0;$ $$-0.424F_B + 0.424F_C = 0 \quad (2)$$

$\Sigma F_z = 0;$ $$0.848F_B + 0.848F_C - 40 = 0 \quad (3)$$

Equation (2) states that $F_B = F_C$. Thus, solving Eq. (3) for F_B and F_C and substituting the result into Eq. (1) to obtain F_D, we have

$$F_B = F_C = 23.6 \text{ lb} \qquad \textit{Ans.}$$

$$F_D = 15.0 \text{ lb} \qquad \textit{Ans.}$$

Example 3

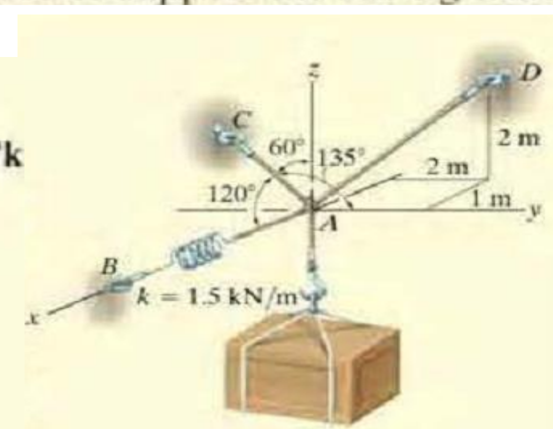
Determine the tension in each cord used to support the 100-kg crate

$$\mathbf{F}_B = F_B\mathbf{i}$$

$$\mathbf{F}_C = F_C \cos 120°\mathbf{i} + F_C \cos 135°\mathbf{j} + F_C \cos 60°\mathbf{k}$$
$$= -0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}}{\sqrt[2]{(-1)^2 + (2)^2 + (2)^2}}\right]$$
$$= -0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k}$$

$$\mathbf{W} = \{-981\mathbf{k}\} \text{ N}$$



Equilibrium requires

$$\Sigma \mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$

$$F_B\mathbf{i} - 0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$
$$- 0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k} - 981\mathbf{k} = \mathbf{0}$$

Equating the respective **i**, **j**, **k** components to zero,

$$\Sigma F_x = 0; \qquad F_B - 0.5F_C - 0.333F_D = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -0.707F_C + 0.667F_D = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad 0.5F_C + 0.667F_D - 981 = 0 \qquad (3)$$

Solving Eq. (2) for F_D in terms of F_C and substituting this into Eq. (3) yields F_C. F_D is then determined from Eq. (2). Finally, substituting the results into Eq. (1) gives F_B. Hence,

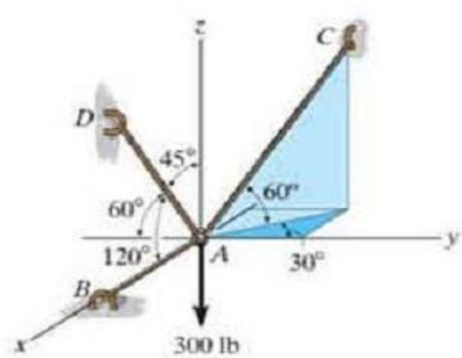
$$F_C = 813 \text{ N} \qquad \textit{Ans.}$$

$$F_D = 862 \text{ N} \qquad \textit{Ans.}$$

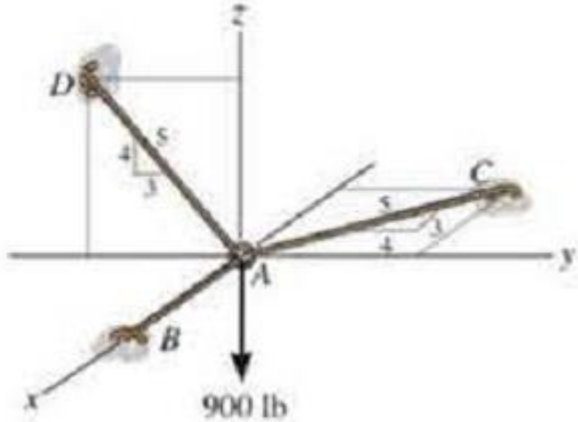
$$F_B = 694 \text{ N} \qquad \textit{Ans.}$$

Example 4

Determine the tension developed in cables AB, AC, and AD.

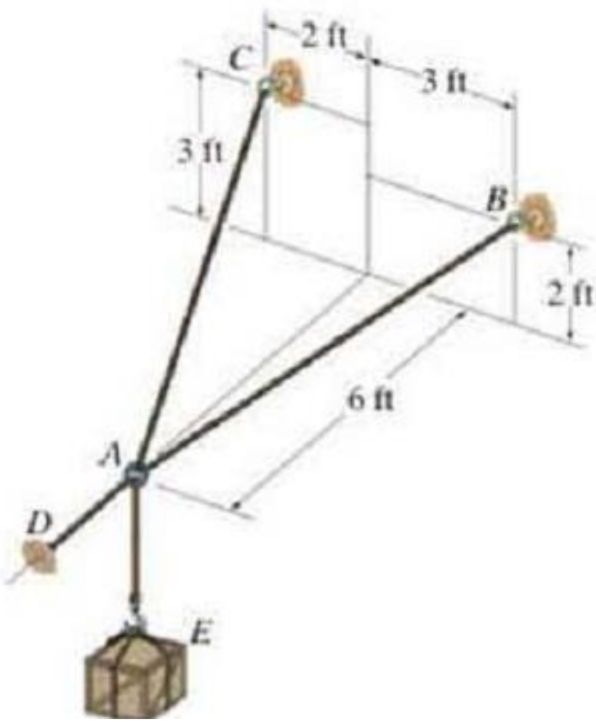


F3–8. Determine the tension developed in cables AB, AC, and AD.

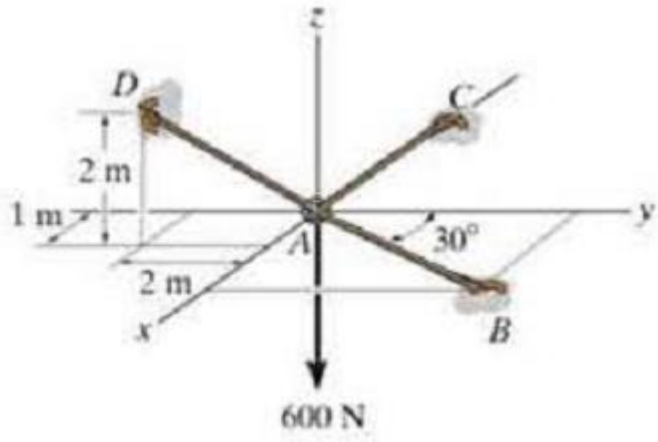


F3–8

F3–11. The 150-lb crate is supported by cables AB, AC, and AD. Determine the tension in these wires.



F3–9. Determine the tension developed in cables AB, AC, and AD.



F3–9